Sample Model Answer – Section 12.1
Biophysics and Physiological Modeling
Chapter 12: COVID-19 and epidemiology (Extended)
Peter Hugo Nelson

Q.12.01(a)

Param$	Variables	Step 0	Step 1
δt (d) $= 1$	t(d)	$t^{\text{new}} = 0$	$t^{\text{new}} = t^{\text{old}} + \delta t$
k_i (1/d) $= 0.3$	R_i (1/d)	–	$R_i^{\text{new}} = k_i * N_i^{\text{old}}$
$N_0 = 10$	N_i	$N_i^{\text{new}} = N_0$	$N_i^{\text{new}} = N_i^{\text{old}} + R_i^{\text{new}} * \delta t$

Unit checks

$$R_i = k_i N_i \,[=]\, \left(\frac{1}{\text{d}}\right)(1) \,[=]\, \frac{1}{\text{d}}$$

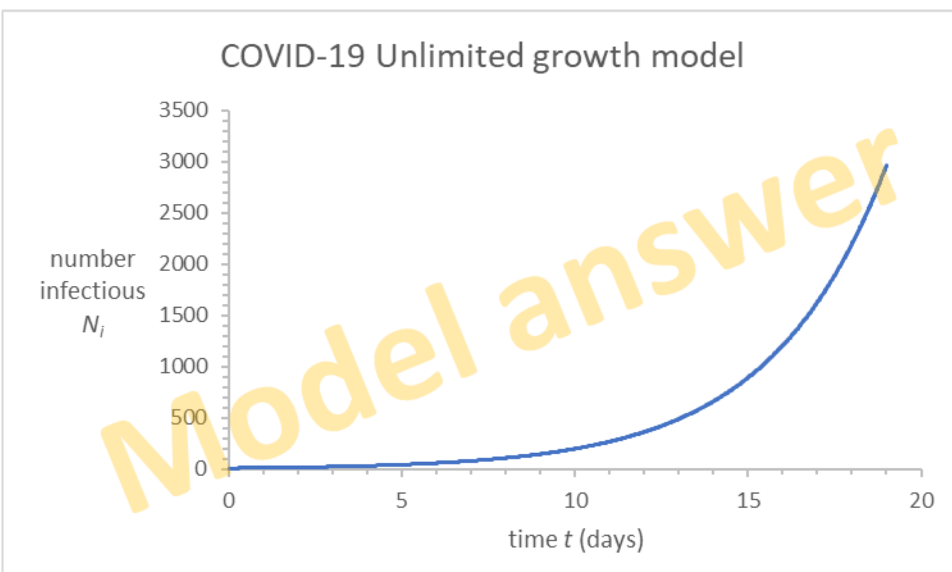
$$N_i^{\text{new}} = N_i^{\text{old}} + R_i^{\text{new}} * \delta t \,[=]\, (1) + \left(\frac{1}{\text{d}}\right)(\text{d}) \,[=]\, 1$$

Q.12.01(b)

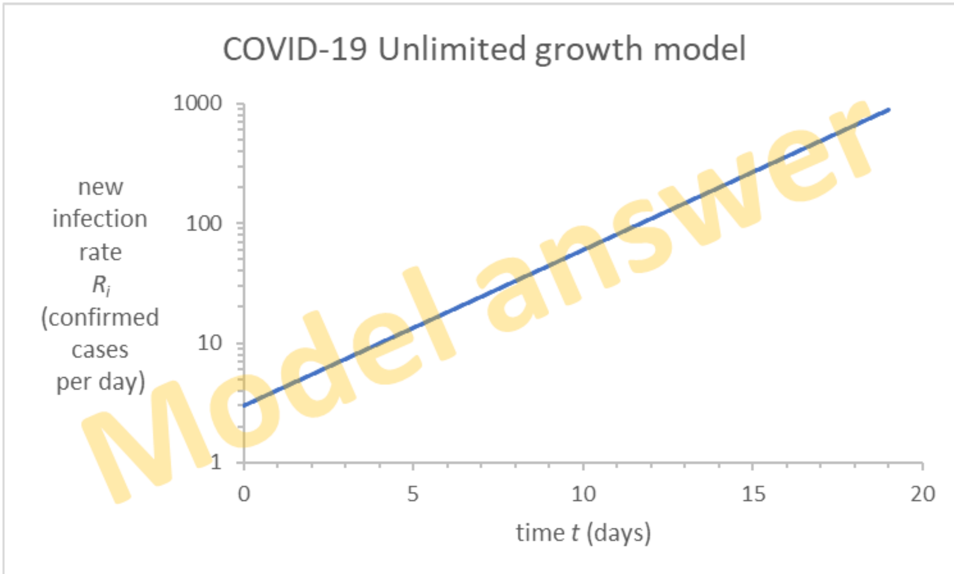
UG model table

time t (d)	rate R_i (1/d)	infectious N_i
0		10
1	3	13
2	3.9	16.9

Q.12.02(a)



Q.12.02(b)



Q.12.02(c) The fact that both graphs are straight lines indicates that they are both exponential functions, i.e. of the form $y = Ae^{bx}$.

Q.12.03(a) Calculus Question:

Q.12.03(b) Calculus Question:

Q.12.04(a) Substituting (12.5) $N_i = N_0 e^{k_i t}$ into (E.1) $R_i = k_i N_i$ yields

$$R_i = k_i N_0 e^{k_i t}$$

Q.12.05(a)

$$N_i = N_0 e^{k_i t}$$

subs $N_i = 2N_0$ and $t = t_d$ yields

$$2N_0 = N_0 e^{k_i t_d}$$

$$2 = e^{k_i t_d}$$

$$\ln 2 = k_i t_d$$

$$t_d = \frac{\ln 2}{k_i}$$

Q.12.05(b)

$$t_d = \frac{\ln 2}{k_i} = \frac{\ln 2}{\left(0.3\,\frac{1}{\mathrm{d}}\right)} = 2.3\ \mathrm{d}$$

Q.12.06(a) On day 30, my spreadsheet (with $\delta t = 0.01$ d) predicted $R_i = 23912$ and $N_i = 79946$.

Q.12.06(b) Equation (12.6) predicts

$$R_i = k_i N_0 e^{k_i t} = \left(0.3\frac{1}{d}\right)(10)\exp\left(\left(0.3\frac{1}{d}\right)(30\ d)\right) = 24309\frac{1}{\mathrm{d}}$$

and equation (12.5) predicts

$$N_i = N_0 e^{k_i t} = 10\exp\left(\left(0.3\frac{1}{d}\right)(30\ d)\right) = 81031$$

The percent error in R_i is

$$\Delta\% = \frac{o-e}{e}\left(\frac{100\%}{1}\right) = \frac{\left(23912\ \frac{1}{\mathrm{d}}\right) - \left(24309\ \frac{1}{\mathrm{d}}\right)}{\left(24309\ \frac{1}{\mathrm{d}}\right)}\left(\frac{100\%}{1}\right) = -1.6\ \%$$

The percent error in N_i is

$$\Delta\% = \frac{o-e}{e}\left(\frac{100\%}{1}\right) = \frac{79946 - 81031}{81031}\left(\frac{100\%}{1}\right) = -1.3\ \%$$

Q.12.06(c) On day 60, my spreadsheet (with $\delta t = 0.01$ d) predicted $R_i = 191{,}169{,}430$ and $I = 639{,}143{,}126$.
Q.12.06(d) Equation (12.6) predicts

$$R_i = k_i N_0 e^{k_i t} = \left(0.3\frac{1}{d}\right)(10)\exp\left(\left(0.3\frac{1}{d}\right)(60\ d)\right) = 196979907\frac{1}{\mathrm{d}}$$

and equation (12.5) predicts

$$N_i = N_0 e^{k_i t} = 10\exp\left(\left(0.3\frac{1}{d}\right)(60\ d)\right) = 656599691$$

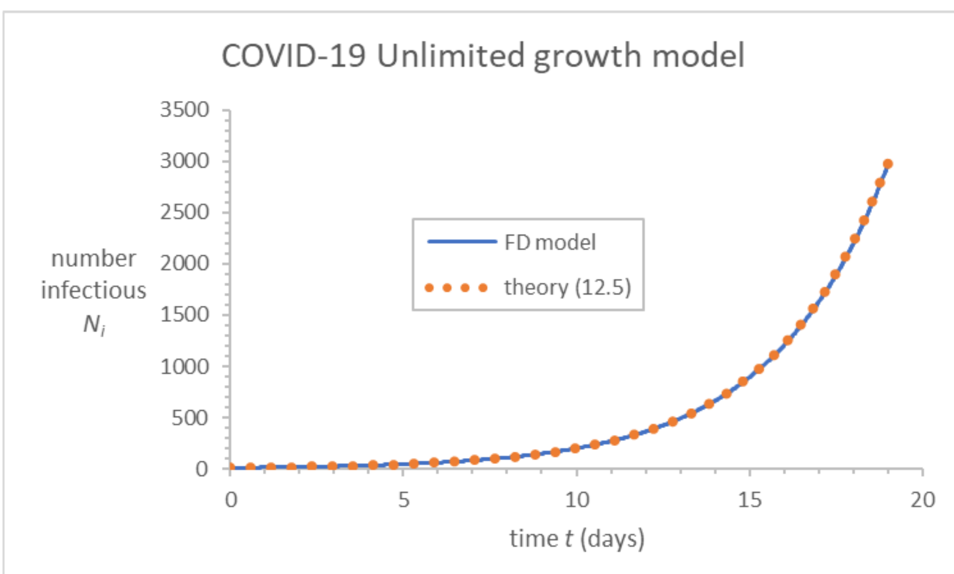
The percent error in R_i is

$$\Delta\% = \frac{o-e}{e}\left(\frac{100\%}{1}\right) = \frac{\left(191169430\ \frac{1}{\mathrm{d}}\right) - \left(196979907\ \frac{1}{\mathrm{d}}\right)}{\left(196979907\ \frac{1}{\mathrm{d}}\right)}\left(\frac{100\%}{1}\right) = -3\ \%$$

The percent error in N_i is

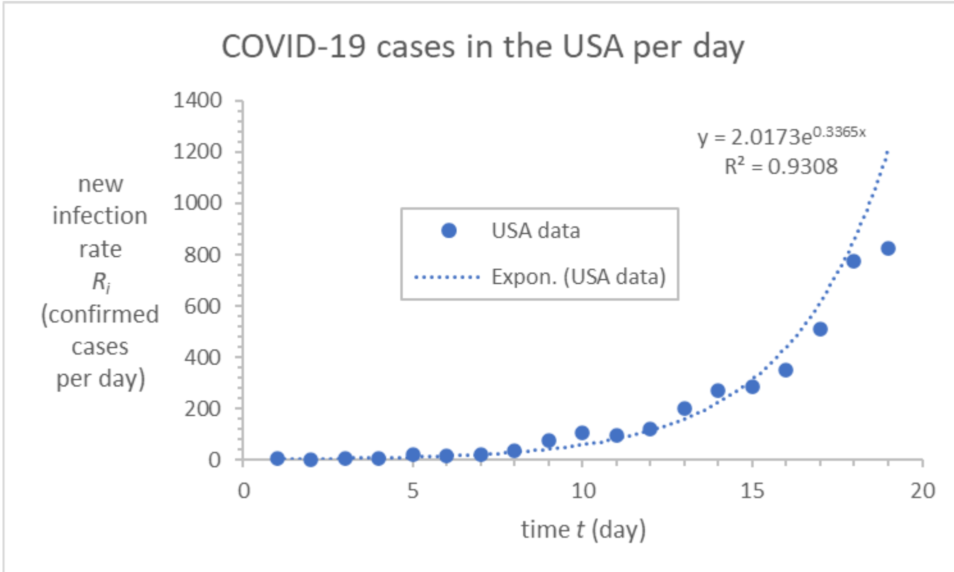
$$\Delta\% = \frac{o-e}{e}\left(\frac{100\%}{1}\right) = \frac{639143126 - 656599691}{656599691}\left(\frac{100\%}{1}\right) = -3\ \%$$

Q.12.07(a) with $\delta t = 0.01$ d, my graph looked like



Q.12.07(b) The FD model and analytical solution appear equivalent if we make δt small enough.

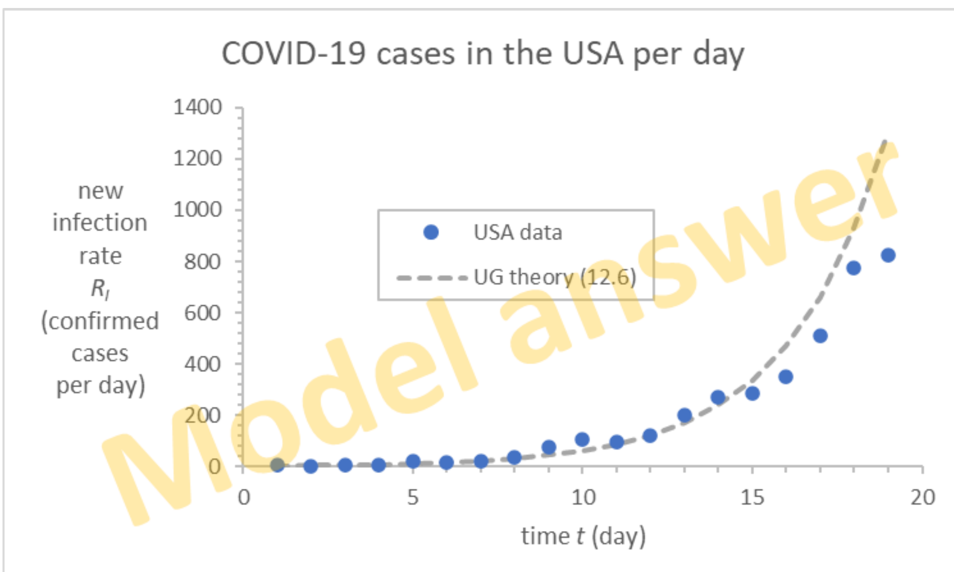
Q.12.08(a)



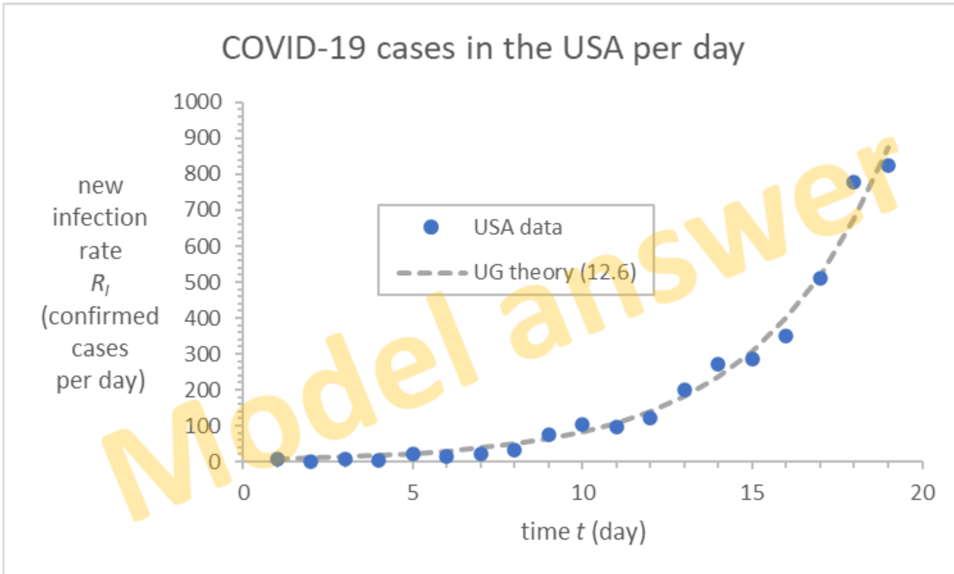
Q.12.08(b) $k_i = 0.3365$ 1/d. $N_0 = 2.0173/k_i = 5.995$.

Q.12.08(c) From the graph for Q.12.8(a), it appears that the USA data are approximately explained by the exponential growth of the UG model, although the last 5 data points appear to be systematically below Excel's exponential trendline.

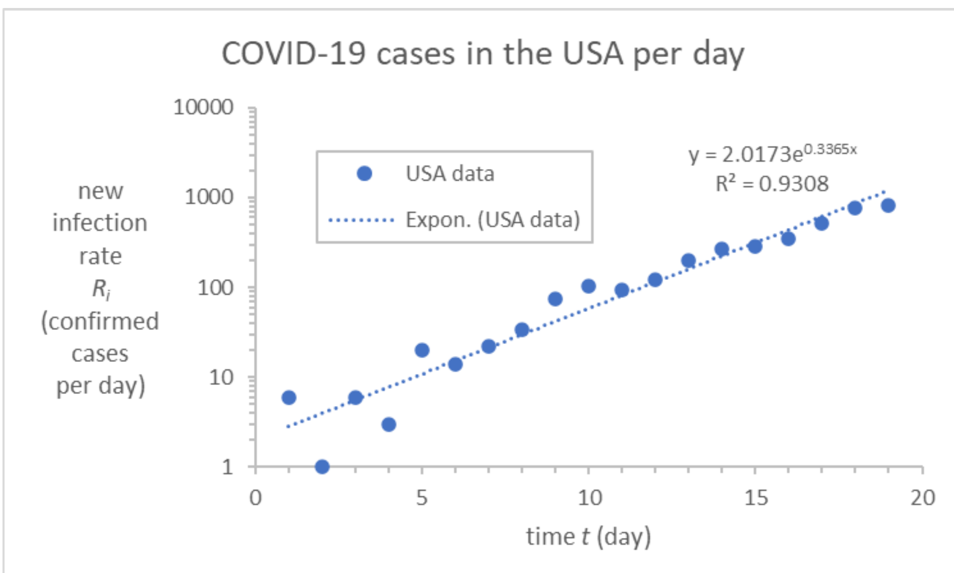
Q.12.09(a)



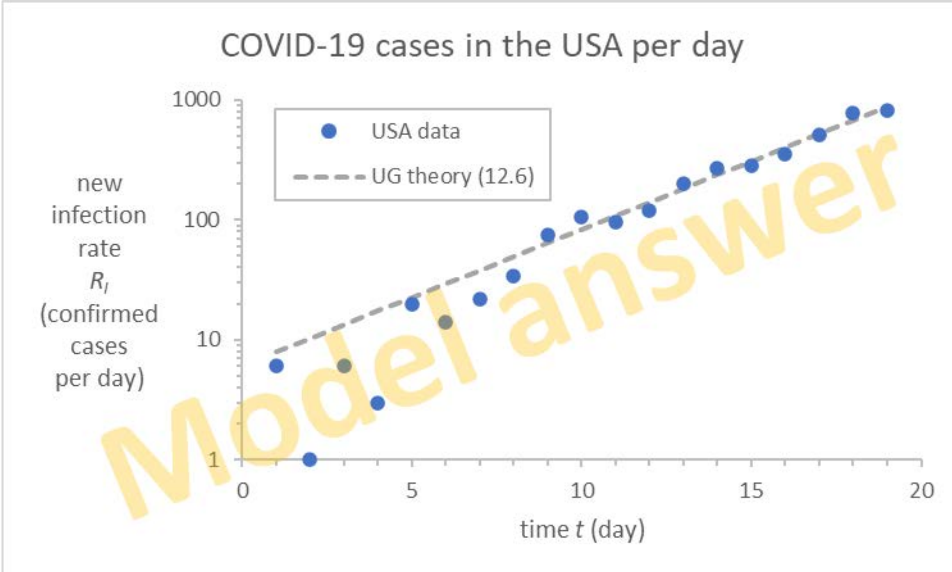
Q.12.09(b)



Q.12.10(a)



Q.12.10(b)



Q.12.10(c) The “exponential trendline” is biased towards small values of R_i, whereas the LS fit treats all data the same in a “least-squares” manner and minimizes the sum of the squares of the residuals.

This material is based upon work supported by the National Science Foundation under Grant No. 0836833. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.